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THE INFLUENCE OF ELASTICITY OF SOME SECTIONS OVER THE EFFORTS FROM THE 4R SPHERICAL SPATIAL MECHANISM

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***Abstract*—**Mechanically speaking (statically), the 4R spherical quadrilateral mechanism is statically undetermined, and for calculating the reactions from the cinematic couples and those influence over the efforts from the mechanism elements ,it is applied the elastic linear calculation using the relative displacements method noted in plukerian coordinates.

In the present paper there will be established the calculation relations that can determine the elasticity influence of every section over efforts from this mechanism.

***Keywords*—** cardan transmission, elastic calculation, pluckerian calculation,.

I. INTRODUCTION

IN the previous paper [1], there was elaborated a mathematical model for the elastic linear calculation of efforts that appear in the 4R spatial spherical mechanism, that is a triple statically undetermined system [2]-[4], [6].

This mechanism presents interest because it reproduces, cinematically speaking, the movement of a system with a cardan joint.

In the present paper are established the connection relations between the geometrical and mechanical characteristics of the sections and efforts that appear during external loads.

II. THEORETICAL ASPECTS

It is considered the 4R mechanism from Fig. 1., in initial position, with the axes concurrent in O, with the rotation couples A, D, fixed and B, C mobile. The fixed reference system $OX_0Y_0Z_0$ is chosen and the θ_1 is the rotation angle of the element AB, θ_2 being the rotation angle of the element CD and α the gradient of axes OD towards the axis OX_0.

For carrying out the elastic linear calculation, the sections $AA', A'A'', A''B, BO, OC, CD'', D''D', D'D$ have been indexed with the numbers $i = 1,2,\ldots 8$ and are considered to have the lengths l_i, $i = 1,2,\ldots 8$, the elasticity modules E_i, G_i, $i = 1,2,\ldots 8$, the sections A_i and the main central inertial moments [7], I_{iy}, I_{iz} , $I_{ix} = I_{iy} + I_{iz}$, $i = 1,2,\ldots 8$.

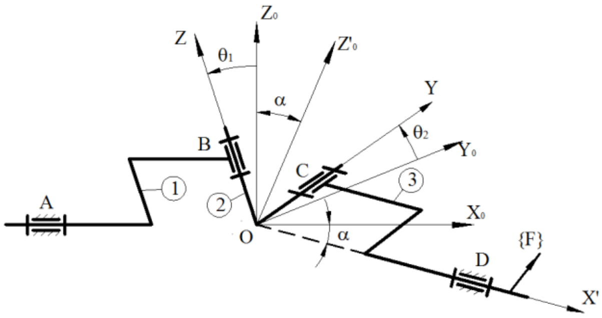

Fig. 1. 4R Asymmetrical spatial spherical mechanism

Notating next with $[H_{AB}]$; $[H_{BC}]$; $[H_{CD}]$ the flexibility matrixes towards the system $OX_0Y_0Z_0$ of the elements AB, BC, CD, with $[H_{AD}]$, $[K_{AD}]$ the flexibility matrixes and the rigidity is defined by the relations

$$[H_{AD}] = [H_{AB}] + [H_{BC}] + [H_{CD}]$$
$$[K_{AD}] = [H_{AD}]^{-1}. \quad (1)$$

With ξ_B, ξ_C, ξ_D the rotation from the cinematic couples (the cinematic couple from A being blocked), $[R^0_{AX}, R^0_{AY}, R^0_{AZ}, M^0_{AX}, M^0_{AY}, M^0_{AZ}]$ the pluckerian coordinates of the reaction from A with the matrixes $\{R^0_A\}, \{\xi\}$

$$\{R^0_A\} = [R^0_{AX}, R^0_{AY}, R^0_{AZ}, M^0_{AX}, M^0_{AY}, M^0_{AZ}]^T$$
$$\{\xi\} = [\xi_B, \xi_C, \xi_D]^T . \quad (2)$$

with : $\{U_B\}, \{U_C\}, \{U_D\}, \{\tilde{U}_B\}, \{\tilde{U}_C\}, \{\tilde{U}_D\}$ the column matrixes attached to the cinematic couples [5]

$$\begin{aligned}
&\{U_B\}=[0,-s\theta_1,c\theta_1,0,0,0]^T\\
&\{\tilde{U}_B\}=[0,0,0,0,-s\theta_1,c\theta_1]\\
&\{U_C\}=[s\theta_2 s\alpha,c\theta_2,s\theta_2 c\alpha,0,0,0]^T\\
&\{\tilde{U}_C\}=[0,0,0,s\theta_2 s\alpha,c\theta_2,s\theta_2 c\alpha]^T\\
&\{U_D\}=[c\alpha,0,-s\alpha,0,0,0]^T\\
&\{\tilde{U}_D\}=[0,0,0,c\alpha,0,-s\alpha]^T.
\end{aligned}\tag{3}$$

where by *c*, *s* where noted the trigonometric functions *cos*, *sin*, and with $[U],[\tilde{U}]$ the matrixes

$$\begin{aligned}
&[U]=[\{U_B\},\{U_C\},\{U_D\}]\\
&[\tilde{U}]=[\{\tilde{U}_B\},\{\tilde{U}_C\},\{\tilde{U}_D\}]
\end{aligned}\tag{4}$$

and with $\tilde{M}$ the moment that actions the axis OD (Fig. 1.) are obtained [5], the results

$$\{\xi\}=-\left[[\tilde{U}][K_{AD}][U]\right]^{-1}[0,0,\tilde{M}]^T.\tag{5}$$

$$\{R_A^0\}=[K_{AD}][U]\{\xi\}.\tag{6}$$

III. CALCULATION RELATIONS

Writing the matrixes $[K_{AD}],[H_{AD}]$ with the help of 3x3 parts

$$\begin{aligned}
&[K_{AD}]=\begin{bmatrix}[K_{11}] & [K_{12}]\\ [K_{21}] & [K_{22}]\end{bmatrix}\\
&[H_{AD}]=\begin{bmatrix}[H_{11}] & [H_{12}]\\ [H_{21}] & [H_{22}]\end{bmatrix}.
\end{aligned}\tag{7}$$

and using the notation

$$[V]=\begin{bmatrix}0 & s\theta_2 s\alpha & c\alpha\\ -s\theta_1 & c\theta_2 & 0\\ c\theta_1 & s\theta_2 c\alpha & -s\alpha\end{bmatrix}.\tag{8}$$

the matrixes $[U],[\tilde{U}]$ are written under:

$$\begin{aligned}
&[U]=\begin{bmatrix}[V]\\ [0]\end{bmatrix}\\
&[\tilde{U}]=\left[[0][V]^T\right]
\end{aligned}\tag{9}$$

and the equality is deduced

$$\left[[\tilde{U}][K_{AD}][U]\right]^{-1}=[V]^{-1}[K_{21}]^{-1}\left[[V]^T\right]^{-1}.\tag{10}$$

so that the relation (5) becomes

$$[\xi]=-\tilde{M}[V]^{-1}[K_{21}]^{-1}\left[[V]^T\right]^{-1}[0,0,1]^T.\tag{11}$$

and the expression (6) is separated into the equalities

$$\begin{aligned}
&\begin{bmatrix}R_{AX}^0\\ R_{AY}^0\\ R_{AZ}^0\end{bmatrix}=-\tilde{M}[K_{11}][K_{21}]^{-1}\left[[V]^T\right]^{-1}[0,0,1]^T\\
&\begin{bmatrix}M_{AX}^0\\ M_{AY}^0\\ M_{AZ}^0\end{bmatrix}=-\tilde{M}\left[[V]^T\right]^{-1}[0,0,1].
\end{aligned}\tag{12}$$

The second relation (12) shows that the moment reactions from A expressed in the $OX_0Y_0Z_0$ system, does not depend of the elements rigidity.

These constituting the components statically determined that can be deduced by elemental balance equations.

The force reactions from A can be deduced with the first relation (12) , that being the forces statically undetermined that depends on the elements rigidity.

The relations (11), (12) can also be written under the form

$$\begin{aligned}
&\begin{bmatrix}M_{AX}^0\\ M_{AY}^0\\ M_{AZ}^0\end{bmatrix}=\frac{\tilde{M}}{s^2\theta_1+c\alpha c^2\theta}\begin{bmatrix}-c\alpha\\ s\alpha s\theta_1 c\theta_1\\ s\alpha s^2\theta_1\end{bmatrix}\\
&\begin{bmatrix}R_{AX}^0\\ R_{AY}^0\\ R_{AZ}^0\end{bmatrix}=-[H_{21}]^{-1}[H_{11}]^T\begin{bmatrix}M_{AX}^0\\ M_{AY}^0\\ M_{AZ}^0\end{bmatrix}.
\end{aligned}\tag{13}$$

$$[\xi]=[V]^{-1}\left[[H_{12}]-[H_{11}][H_{21}]^{-1}[H_{22}]\right]\begin{bmatrix}M_{AX}^0\\ M_{AY}^0\\ M_{AZ}^0\end{bmatrix}.\tag{14}$$

Expressing the relation (13), (14) by using flexibility matrixes, allows the analysis of elasticity on components, knowing that for the rigid components the flexibility matrixes are null.

In order to have the first image over these influences it will be analyzed first the case where:

$\alpha=0^\circ;\theta_1=0^\circ$, case where results as expected $M_{AX}^0=-\tilde{M}$, $M_{AY}^0=M_{AZ}^0=0$.

It also results that

$$[V]=\begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}.\tag{15}$$

so,

$$\begin{bmatrix} R_{AX}^0 \\ R_{AY}^0 \\ R_{AZ}^0 \end{bmatrix} = \tilde{M}\left[H_{21}\right]^{-1}\left[H_{22}^*\right]\left[0,0,1\right]^T \tag{16}$$

$$[\xi] = -\tilde{M}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\left[\left[H_{12}\right]-\left[H_{11}\right]\left[H_{21}\right]^{-1}\left[H_{22}\right]\right]\left[0,0,1\right].$$

IV. NUMERICAL APPLICATION

It is considered the case where sections are identical, by having:
$l_i = l = 0{,}04m$; $i = 1,2,\ldots 8$; $d_i = d = 0{,}03m$; $i = 1,2,\ldots 8$;
$E = 2{,}1\cdot 10^{11} daN/m$; $G = 2{,}1\cdot 10^{10} daN/m$;
$\tilde{M} = 10Nm$.

The calculus of flexibility matrixes was made after the previous calculation from paper [5] and the results are presented in TABLE I for $\alpha = 0^\circ$; $\theta_1 = 0^\circ$.

In Fig. 2. and Fig. 3., the results are transcribed in diagrams for the second case where the section 2 is elastic and $\alpha = 0^\circ$; $\theta_1 \in \left[0^\circ, 360^\circ\right]$.

For the last two cases $\alpha = 10^\circ$; $\theta_1 \in \left[0^\circ, 360^\circ\right]$ the results are transcribed in the diagrams from Fig. 2., Fig. 3., Fig. 4. and Fig. 5.

TABLE I
ROTATION ANGLE AND REACTION FORCE

Rigid sections no.	Elastic sections no.	ξ_B [radians]	ξ_C [radians]	ξ_D [radians]	R^0_{AX} [N]	R^0_{AY} [N]	R^0_{AZ} [N]
2,3,4,5,6,7,8	1	0	0	$-6{,}2\cdot 10^{-5}$	0	0	0
1,3,4,5,6,7,8	2	$2{,}0\cdot 10^{-5}$	0	$-4{,}0\cdot 10^{-5}$	0	76,7	0
1,2,4,5,6,7,8	3	$1{,}9\cdot 10^{-5}$	0	$-1{,}3\cdot 10^{-5}$	0	198,8	0
1,2,3,5,6,7,8	4	0	0	$-1{,}2\cdot 10^{-5}$	0	375,0	0
1,2,3,4,6,7,8	5	0	0	$-1{,}2\cdot 10^{-5}$	0	-375,0	0
1,2,3,4,5,7,8	6	0	$-1{,}9\cdot 10^{-5}$	$-1{,}3\cdot 10^{-5}$	0	0	-198,8
1,2,3,4,5,6,8	7	0	$-1{,}9\cdot 10^{-5}$	$-4{,}0\cdot 10^{-5}$	0	0	-76,7
1,2,3,4,5,6,7	8	0	0	$-6{,}2\cdot 10^{-5}$	0	0	0
1,4,5,8	2,3,6,7	$3{,}0\cdot 10^{-5}$	$-3{,}0\cdot 10^{-5}$	$-14\cdot 10^{-5}$	84,2	115,2	-115,2
-	1,2,3,4,5,6,7,8	$1{,}8\cdot 10^{-5}$	$-1{,}8\cdot 10^{-5}$	$-38\cdot 10^{-5}$	36,4	63,2	63,2

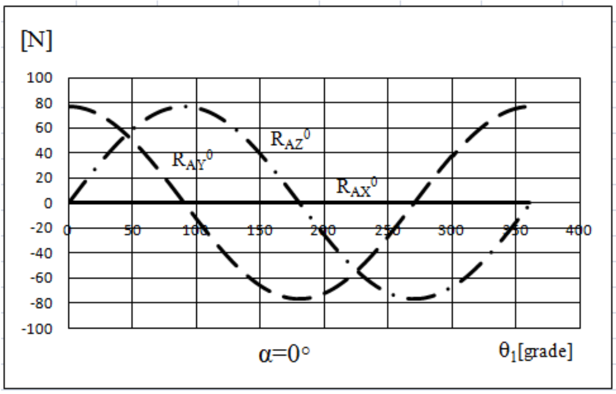

Fig. 2. Reaction forces variation in the general reference system in joint A, when the section 2 is elastic and α=0°.

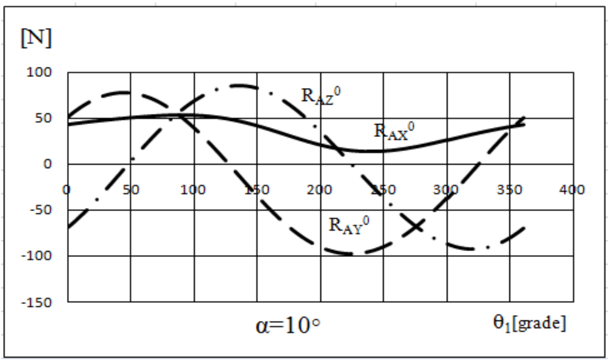

Fig. 4. Reaction forces variation in the general reference system in joint A, when the sections 2, 3, 6 and 7 are elastic and α=10°.

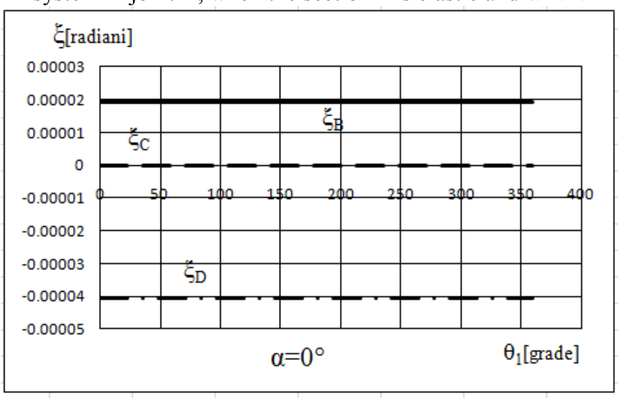

Fig. 3. The variation of relative rotations in joints when section 2 is elastic and α=0°.

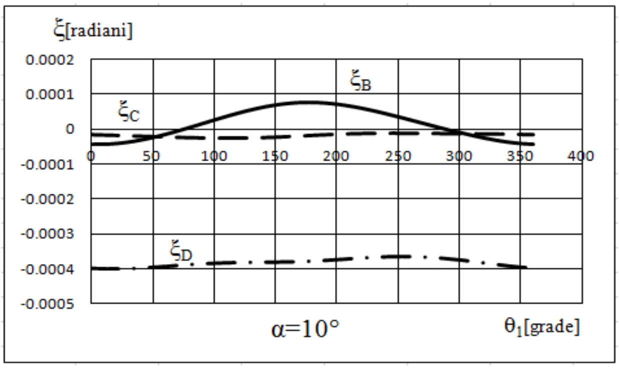

Fig. 5. The variation of relative rotations in joints when sections 2, 3, 6 and 7 are elastic and α=10°.

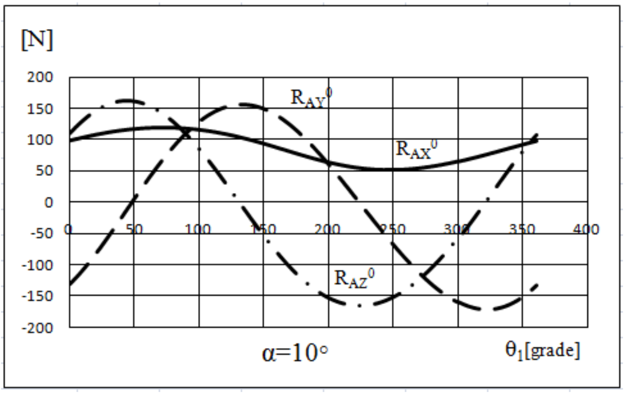

Fig. 6. Reaction forces variation in the general reference system in joint A, when all sections are elastic and α=10°.

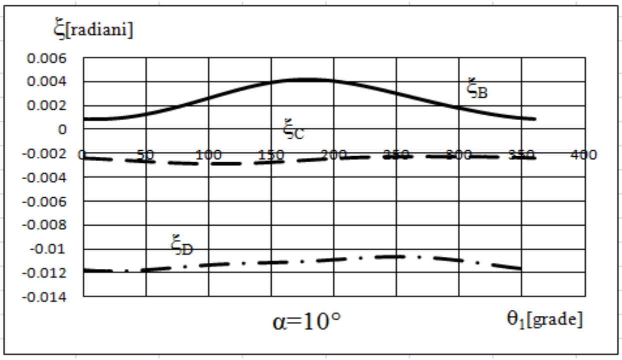

Fig. 7. The variation of relative rotations in joints when all sections are elastic and α=10°

V. CONCLUSIONS

The mathematical model established in paper [1] and the program that was made, allows the user to determine the influence of elasticity of each element of the spherical 4R mechanism over the reactions from its cinematic couples. Following those numerical simulations, by consecutive stiffening of the component sections, it is noticed that some elastic combination of elements conduct to an axial force F_{AX}^0 for $\alpha = 0^\circ ; \theta_1 = 0^\circ$.

As can one notice from diagrams from Fig. 2. and Fig. 3., for the chosen rigidity model, the axial force F_{AX}^0 is zero for the entire domain $\theta_1 \in \left[0^\circ, 360^\circ\right]$, and the distinctive rotations ξ_B , ξ_C , ξ_D , are constant.

The size of forces $F_{AX}^0, F_{AY}^0, F_{AZ}^0$, for the same external loads, depends on the mechanical and geometrical characteristics of the sections, also on the way they are combined as can be deduced from the first relation (16) and the presented diagrams.

By using this method and numerical calculation, allows one to determine the reactions from the cinematic couples, as the method can be applied to different types of spatial or plane mechanisms.

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